Multirobot Simultaneous Path Planning and Task Assignment on Graphs with Stochastic Costs

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Introduction

Given:

- Initial positions of robots s_i and tasks t_i;
- A graph $G(V, E)$ that captures the collision-free configuration space of the robots with respect to static obstacles;
- Means ${}^i\mu_{uv}$ and variances ${}^i\sigma^2_{uv}$ of uncertain travel cost ${}^ic_{uv}$ of any robot r_i moving on any edge e_{uv} on graph.

Compute:

- Assignment of robots to tasks z_{ij} and paths to reach the tasks $\{{}^ix_{uv}\}$;
- Minimize a cost value y such that cost of every robot is less than y with high probability p in any realization of ${}^ic_{uv}$.

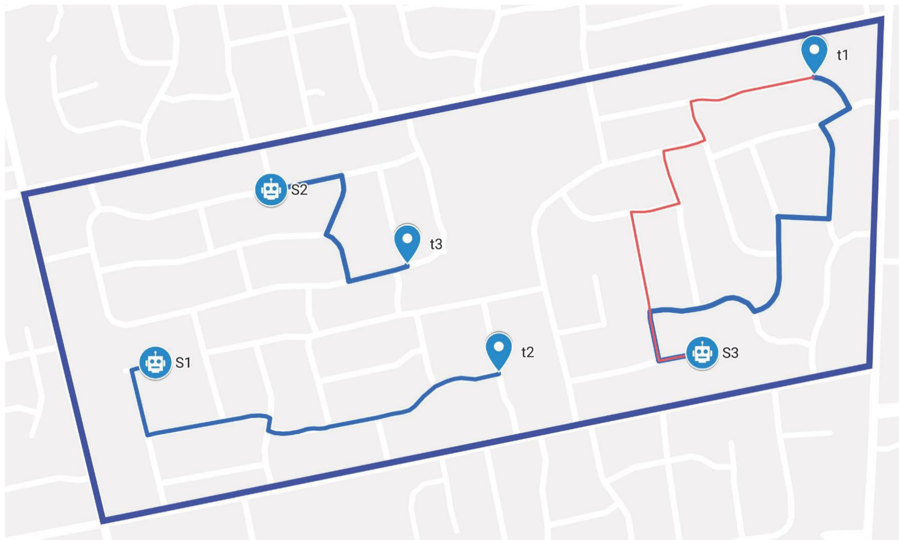

Fig. 1. Simultaneous task allocation and path planning for robots with stochastic travel costs.

Problem formulation

Chance-constrained simultaneous task assignment and planning problem (CC-STAP).

$$\begin{aligned}
&\min \quad y \\
&\text{s.t.} \quad \mathbb{P}\left(\sum_{u=1}^{N}\sum_{v=1}^{N} {}^ic_{uv}\,{}^ix_{uv} \le y\right) \ge p, \quad i = 1,\dots,n. \\
&\sum_{v\in\mathcal{N}(u)} {}^ix_{uv} - \sum_{v\in\mathcal{N}(u)} {}^ix_{vu} = \begin{cases} 1, & \text{if } u = s_i, \\ -z_{ij}, & \text{if } u = t_j, \\ 0, & \forall u \in V \setminus \{s_i, t_j\} \end{cases} \\
&\qquad i = 1,\dots,n. \\
&\sum_{i=1}^{n} z_{ij} = 1,\ \forall j; \quad \sum_{j=1}^{n} z_{ij} = 1,\ \forall i. \\
&{}^ix_{uv} \in \{0,1\}, \quad z_{ij} \in \{0,1\}.
\end{aligned} \tag{1}$$

- The solution includes both assignment z_{ij} and path to reach the assignment ${}^ix_{uv}$. Every robot should be assigned to a unique task and compute a path from source s_i to its task.
- Chance constraints provide a probabilistic quality guarantee, which ensures that the travel cost for any robot is less than y with probability at least p.
- This problem is a non-convex integer optimization problem which is difficult to solve directly.

Contributions

- We prove that the problem can be solved by solving multiple related subproblems: chance-constrained shortest path problems (CC-shortest path) for all robot-task pairs, linear bottleneck assignment problem (LBAP).
- We develop a novel two-step algorithm for CC-STAP that
  1. Solves CC-shortest path problems for all robot-task pairs.
  2. Solve LBAP with l_{ij} equal to the optimal objective of CC-shortest path for robot r_i to task t_j.

Chance-constrained shortest path

- CC-shortest path problem for a robot-task pair (r_i, t_j)

$$\begin{aligned}
&\min \ {}^iy_j \\
&\text{s.t.} \ \sum_{u,v} {}^i\mu_{uv}\,{}^ix_{uv} + C\sqrt{\sum_{u,v} {}^i\sigma^2_{uv}\,{}^ix_{uv}} \le {}^iy_j \\
&\sum_{v\in\mathcal{N}(u)} {}^ix_{uv} - \sum_{v\in\mathcal{N}(u)} {}^ix_{vu} = \begin{cases} 1, & \text{if } u = s_i, \\ -1, & \text{if } u = t_j, \\ 0, & \forall u \in V \setminus \{s_i, t_j\} \end{cases} \\
&{}^ix_{uv} \in \{0,1\}
\end{aligned} \tag{2}$$

- The solution is a path for r_i from the source to task t_j whose cost for path should be less than cost value ${}^iy_j$ with probability at p irrespective of the travel cost realizations. Let the optimal ${}^iy_j$ be $l_{ij} = {}^iy_j*$.
- The first constraint is equivalent to the chance constraint in (1) for r_i with cost value equal to ${}^iy_j$. $C = \Phi^{-1}(p)$ if the random travel cost is Gaussian. $C = \sqrt{\frac{p}{1-p}}$ for case where the probability distribution is unknown.

Linear bottleneck assignment

- The assignment in (1) can be obtained by solving a LBAP.

$$\begin{aligned}
&\min \ \max_i \sum_j \ell_{ij} z_{ij} \\
&\text{s.t.} \ \sum_i z_{ij} = 1, \ \sum_j z_{ij} = 1, \ z_{ij} \in \{0,1\}\ \forall i, j.
\end{aligned} \tag{3}$$

- The constraints are same as the assignment constraints in (1) that ensure each robot performs a unique task.
- l_{ij} is equal to the optimal objective of (2) for pair (r_i, t_j)
- This is a well-studied problem which can be solved by threshold algorithm once l_{ij} are computed.

Geometric interpretation of CC-SP

- Any path is a point on a 2D variance-mean plane where x-axis, y-axis represent the sum of variances and means of the edges in the path.
- The optimal path is an extreme point of feasible point set.

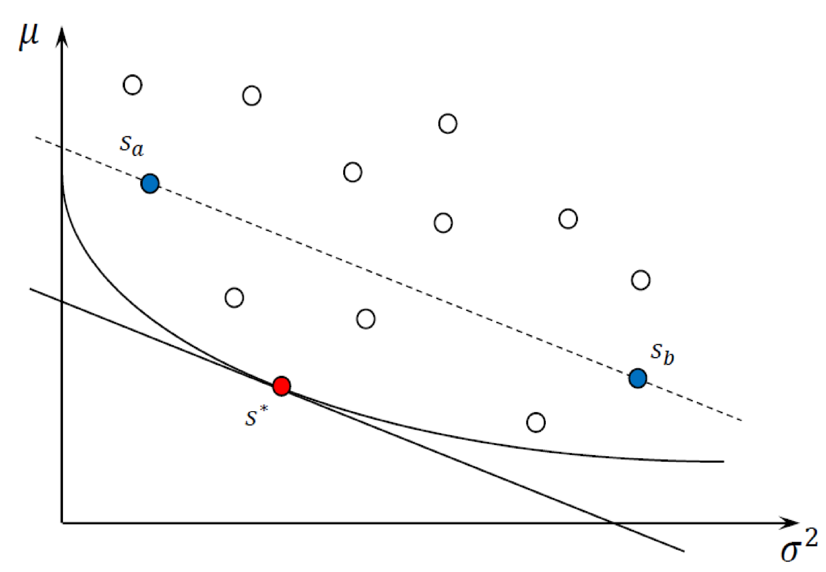

Fig. 2. Variance-mean plane. Each feasible path is treated as a point and the optimal path is an extreme point of the solution set.

The optimal CC-SP solution is also optimal for a deterministic shortest path problem with certain value of λ. The edge cost is a linear combination of mean and variance of random cost.

$$\begin{aligned}
&\min \sum_{u,v} ({}^i\mu_{uv} + \lambda\,{}^i\sigma^2_{uv})\,{}^ix_{uv} \\
&\text{s.t.} \ \sum_{v\in\mathcal{N}(u)} {}^ix_{uv} - \sum_{v\in\mathcal{N}(u)} {}^ix_{vu} = \begin{cases} 1, & \text{if } u = s_i, \\ -1, & \text{if } u = t_j, \\ 0, & \forall u \in V \setminus \{s_i, t_j\} \end{cases} \\
&{}^ix_{uv} \in \{0,1\}
\end{aligned} \tag{4}$$

- The problem becomes a one-dimensional search on risk-averse parameter λ.

Algorithm for CC-shortest path

1. Solve a sequence of deterministic shortest path problems with methodically increasing λ (starting from 0). The updating rule is $\lambda_{k+1} = c/\sigma_k$. The upper bound for optimal λ is obtained once $\lambda_k\sigma_k = C$.
2. Search for the optimal λ^* by enumerating the extreme points in the obtained search region by solving a sequence of deterministic problems with λ generated by computing the slope of lines connecting any consecutive obtained points.

Simulation results

- We perform simulations to study the scalability of our algorithm to the number of the robots (Fig. 3) and the size of the graph (Fig. 4). We count the total number of the deterministic problems solved for a STAP which influences the efficiency.
  - The average and maximum number are computed from 100 scenarios with randomly generated means, variances (Fig.3, Fig. 4) and graphs (Fig. 4).
- An approximate solution obtained from the first step of the algorithm is also studied. In Fig. 3 the approximate solution is greater than the optimal solution by at most 0.22% and 0.19% in Fig. 3 and Fig. 4 respectively.

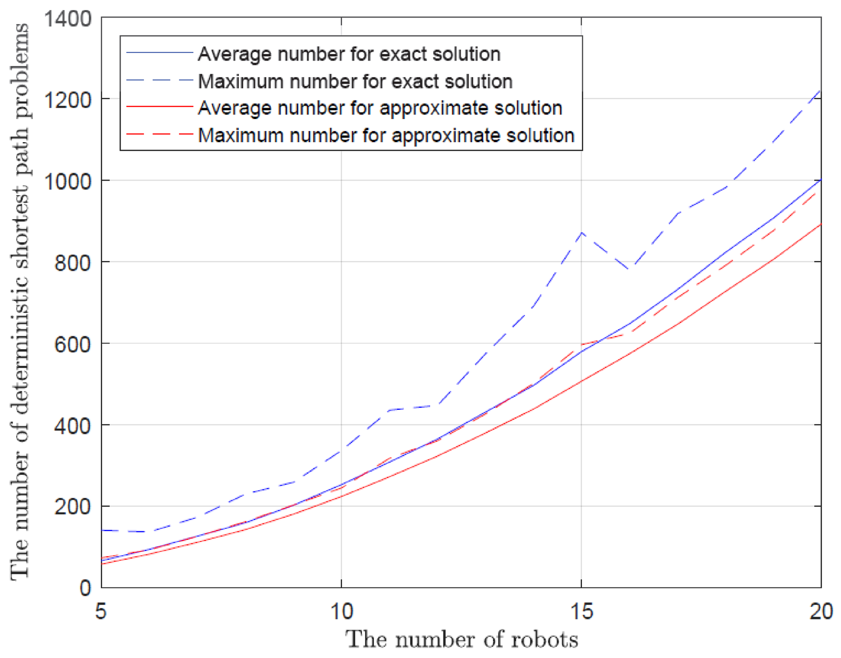

Fig. 3. The deterministic problems solved with respect to the number of robots.

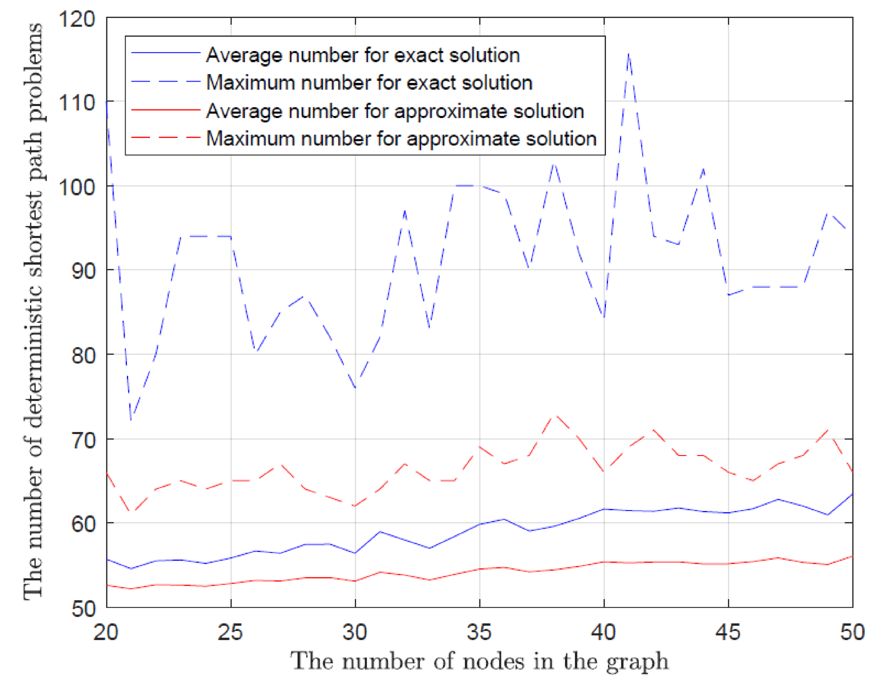

Fig. 4. The deterministic problems solved with respect to the number of nodes in graph.

Conclusion

- We formulate a CC-STAP that consider both task allocation and path planning with the stochastic variables.
- CC-STAP can be decomposed into a set of CC-shortest path problems and a LBAP.
- CC-shortest path problem can be solved by solving a sequence of deterministic shortest path problems.

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