Multi-Robot Team Formation Under Uncertain Environment

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Introduction

Multi-robot team formation under uncertain environment arises in a wide variety of application scenarios like patrolling, automated transport of goods, environmental monitoring and surveillance.

We consider a pipeline surveillance problem with a team of quadrotors (see Fig.1). Given a route of a pipeline with known distance (L in Eq.1) and a set of quadrotors with different operation cost (c_i in Eq.1) and ability of travelling (l_i in Eq.1) which is random variable, we need to form a team of quadrotors from the given set to detect leakage such that in any realization the quadrotor team covers the pipeline with at least a pre-specified probability while the overall operation cost is minimized.

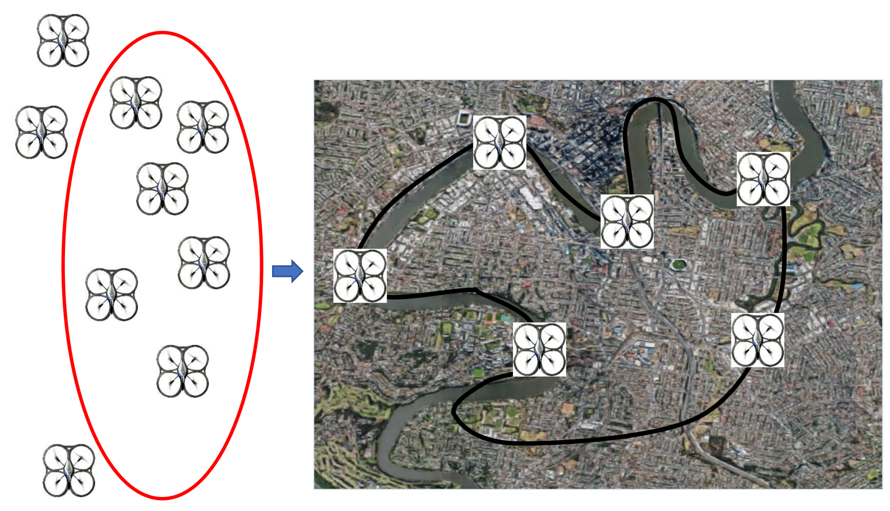

Fig. 1. A pipeline should be traversed by robots with random travel distances. The goal is to select a team of robots from the given set to minimize the total cost such that the route is traversed with probabilistic guarantee.

Problem Formulation

This problem is a chance-constrained knapsack problem (CC-KAP):

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} c_i f_i \\ \text{s.t.} \quad & \mathbb{P}\left(\sum_{i=1}^{n} \ell_i f_i \geq L\right) \geq p \qquad (1) \\ & f_i \in \{0,1\}, \quad \forall i = 1,\ldots,n \end{aligned}$$

- The solution is a vector where each entry f_i is a binary decision variable indicating that a robot is selected when $f_i = 1$.
- The chance constraint guarantees that under any realization of the random travel distance, the selected team covers the route with at least a pre-specified probability p.
- The chance constraint is equivalent to a deterministic constraint $\sum_i \mu_i f_i - C\sqrt{\sum_i \sigma_i^2 f_i} \geq L$ where C is a constant determined by p.

To solve CC-KAP, we solve a sequence of deterministic problems, called risk-averse knapsack problem (RA-KAP).

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} c_i f_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} \mu_i f_i - \lambda \sum_{i=1}^{n} \sigma_i^2 f_i \geq L' \qquad (2) \\ & f_i \in \{0,1\}, \quad \forall i = 1,\ldots,n \end{aligned}$$

This problem is essentially a deterministic 0-1 knapsack problem where the travel distances of robots are linear combinations of means (μ_i) and variances (σ_i^2), λ is risk-averse parameter and the route length is L'.

Geometric Interpretation

- The problem is analyzed on a 2D variance-mean plane (see Fig.2) where x-axis, y-axis represent the sum of variances and means of the selected robots respectively.
- Any robot team can be represented as a point on variance-mean plane.
- The optimal solution is the point in the non-convex feasible region (above blue parabola in Fig.2a) with the highest objective value.
- The optimal solution of CC-KAP is the optimal solution of a RA-KAP with appropriate choice of (λ, L').
- Thus, the problem is converted to a two-dimensional search (λ, L') on variance-mean plane.

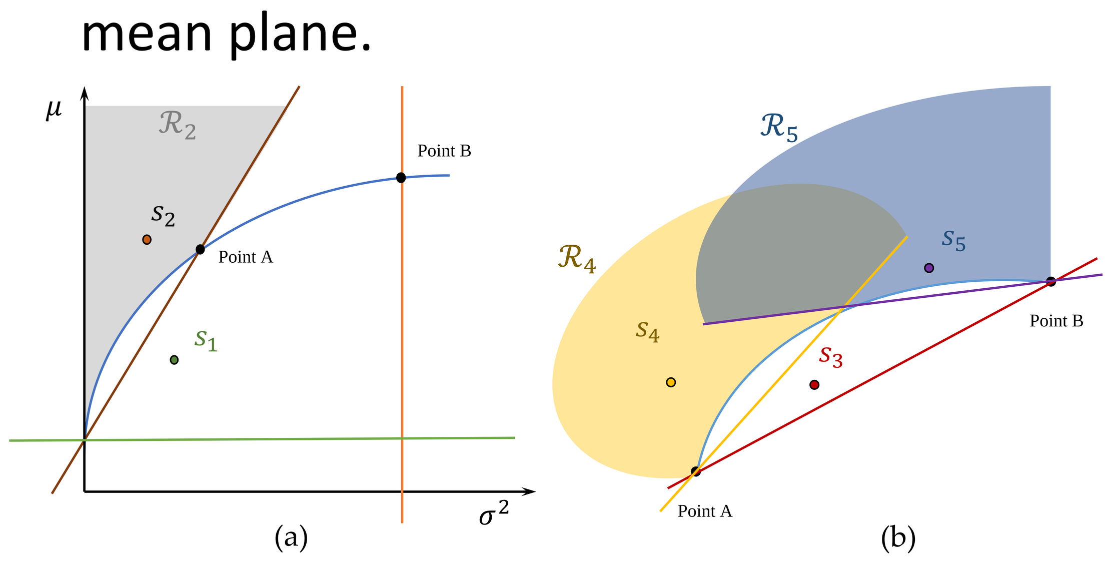

Fig. 2. Illustration of our two-step algorithm. The feasible region of CC-KAP ($\mathcal{R}$) is the non-convex space above the parabola. Fig.(a) illustrates the first step, where we find a feasible solution of CC-KAP (s_2) which is optimal in the intersection region $\mathcal{R}_2$. Fig.(b) zooms in the remaining feasible region $\mathcal{R}\backslash\mathcal{R}_2$. It illustrates the second step, where we find several feasible solutions, (s_4, s_5) that are optimal in the intersection region of CC-KAP and the corresponding RA-KAP, ($\mathcal{R}_4$, $\mathcal{R}_5$) such that the union of those intersection regions and the region in the first step, $\mathcal{R}_2$ covers the feasible region of CC-KAP, e.g. $\mathcal{R} \subseteq \cup_{i\in\{2,4,5\}} \mathcal{R}_i$.

Algorithm

We develop a two-step algorithm to solve the CC-KAP optimally:

1. Solve a sequence of RA-KAPs by methodically increasing λ that controls the slope of the straight line until the optimal solution of a RA-KAP (s_2 in Fig. 2a) is feasible to CC-KAP.
2. Solve RA-KAPs by methodically changing parameters (λ, L') that control the slope and y-intercept of the straight line. If the optimal solution of a RA-KAP is feasible to CC-KAP, we obtain a solution that is optimal in the intersection of feasible regions of CC-KAP and RA-KAP, e.g., $\mathcal{R}_4$, $\mathcal{R}_5$ in Fig.2b. The *procedure terminates* when the intersection regions corresponding to those feasible solutions, e.g., $\mathcal{R}_4$, $\mathcal{R}_5$, cover the remaining feasible region of CC-KAP at the end of the first step, i.e., $\mathcal{R}\backslash\mathcal{R}_2$, in Fig.2a.

Simulation Results

Simulations are performed based on randomly generated means and variances for travel distance capacity of each robot. We count the number of RA-KAPs solved which influences the efficiency and scalability of our algorithm.

- Fig. 3 shows the scalability of our algorithm as a function of the number of robots varying from 10 to 100. The results shown are obtained from 100 randomly generated scenarios.
  - The average numbers of RA-KAP solved is constant (< 3) irrespective of the number of robots while the maximum number is at most 7 (see Fig.3).
- Our algorithm is scalable with the number of robots or variances of travel distances (not shown in poster).

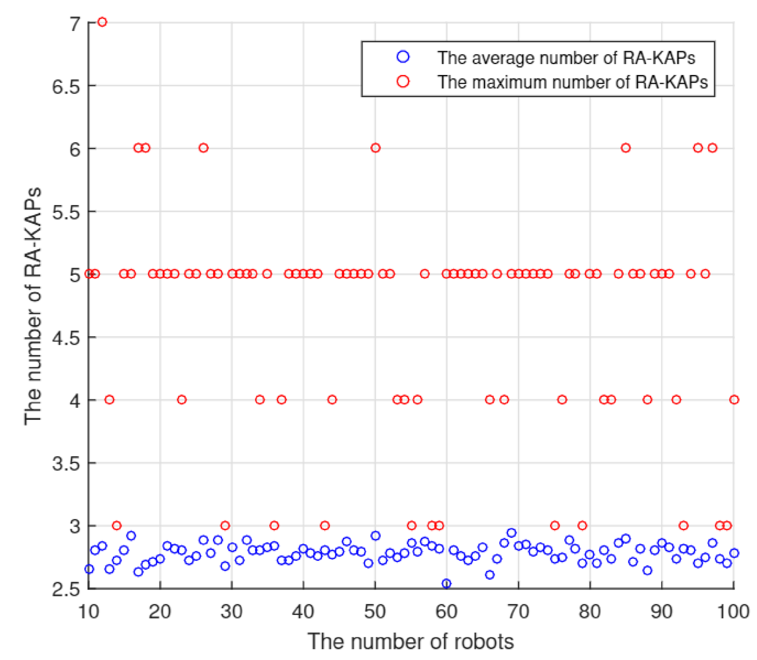

Fig. 3. For a given number of robots (varying from 10 to 100), the average and maximum number of RA-KAPs solved are less than 3 and 7 respectively. Results are based on 100 simulations with randomly generated mean and variance for travel distance of each robot.

Conclusion

- We present a novel approach that uses the solutions of a small number of deterministic RA-KAPs to solve a multi-robot team formation problem optimally.
- We analyze the relationship between CC-KAP and RA-KAP on variance-mean plane. The geometric insight is helpful for solving other problems with chance constraint.
- We present simulation results showing that our method is efficient and scalable with the number of robots and the uncertainty inn travel distance capacity.

Acknowledgements

This work was supported in part by AFOSR award FA9550-15-1-0442.